DOI: 10.24904/footbridge2017.09424

# VERTICAL ACCELERATIONS DUE TO JOGGERS OF A SHORT SPAN FOOTBRIDGE

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## 1. Introduction

Nowadays more and more slender footbridges are designed. Footbridges are designed thinner to increase aesthetic value, as well as incorporate sustainability and to reduce building costs. The consequence is that the eigenfrequency of the bridge can drop so low that it may fall into the range of frequencies that is susceptible to vibrations caused by human beings who move along the bridge. When the eigenfrequency of the bridge is nearly the same as the step frequency of a person who moves along the footbridge this may lead to vertical accelerations exceeding the allowed limits.

Footbridges are calculated according to the Eurocodes. The National Annex of the Eurocode 1990 refers to the guideline EUR 23984 EN “Design of Lightweight Footbridges for Human Induced Vibrations”. In this guideline calculation methods are described to determine the expected accelerations in vertical direction, while limitations are given to the maximum occurring accelerations in the vertical direction. In short a harmonic oscillating point load, representing joggers, is applied stationary at midspan of the footbridge. The vertical accelerations after a theoretically infinite time frame, the steady-state, are taken into account.

One might question whether this is a realistic approximation of what actually happens in practice. In particular with short span footbridges joggers will be present on the footbridge for just a short period of time. In addition the joggers move across the footbridge, so the bridge will only be excited a fraction of time.

An analytical model is proposed in which a jogger is modelled as a harmonic oscillating point load moving forward in time.

The calculated vertical accelerations of this analytical model will be compared to the calculated vertical accelerations obtained by a model prescribed in the guideline EUR 23984 EN.

The expected accelerations of the footbridge being reviewed, are the accelerations resulting from the steady-state calculated according to the Single Degree of Freedom Method (SDOFM).

## 2. Analytical model

In case of a one span statically determined beam footbridge with a short span, the vertical accelerations at midspan due to joggers calculated according to the SDOFM exceed the maximum allowed accelerations. This is true for almost all cases in which the eigenfrequency of the footbridge falls in the critical range of step frequencies.

The proposed analytical model uses a harmonic oscillating point load moving forward in time, representing a jogger crossing a footbridge that is a one span statically determined beam. The vertical accelerations are calculated at midspan. The analytical model is given in Fig. 1.

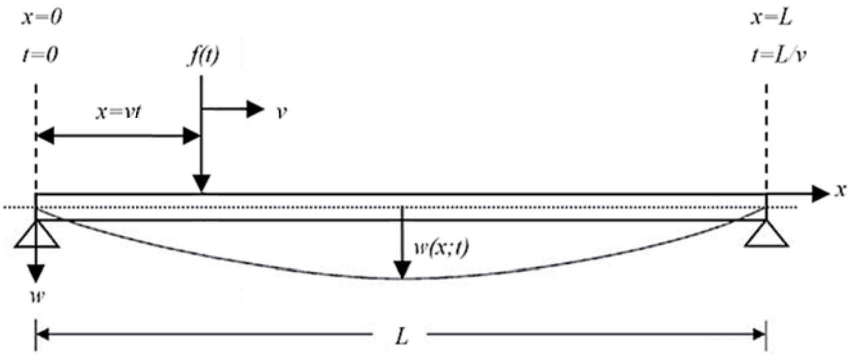


*Fig. 1. Analytical model*

The analytical model can be written as an equation of motion, see (1).

$$\rho A\ddot{w}(x;t) + c\dot{w}(x;t) + EIw''''(x;t) = f(t)\delta(x-vt)[H(t)-H(t-L/v)] \qquad (1)$$

Solving the differential equation results in a formula for determining the displacements of the footbridge at midspan in the course of time. A distinction is made between the phase where joggers are present on the footbridge and the phase where joggers are no longer present on the footbridge, see (2) and (3).

$$w(x;t) = \frac{2}{\rho AL}\sum_{n=1}^{\infty}\frac{Y_n(x)}{\omega_n\eta}\int_0^t P\sin(\Omega\tau)\,Y_n(v\tau)e^{-\frac{b}{2}(t-\tau)}\sin\big(\omega_n\eta(t-\tau)\big)\,d\tau \qquad \text{voor} \quad 0 < t \leq L/v \qquad (2)$$

$$w(x;t) = \frac{2}{\rho AL}\sum_{n=1}^{\infty}\frac{Y_n(x)}{\omega_n\eta}\int_0^{L/v} P\sin(\Omega\tau)\,Y_n(v\tau)e^{-\frac{b}{2}(t-\tau)}\sin\big(\omega_n\eta(t-\tau)\big)\,d\tau \qquad \text{voor} \quad t > L/v \qquad (3)$$

The accelerations of the footbridge at midspan can be calculated by taking the second derivative of the formulas given in (2) and (3) with respect to time. These results are compared with the results of the accelerations calculated according to the SDOFM prescribed in guideline EUR 23984 EN.

## 3. Case Study

Two experiments are conducted at an existing footbridge in which the occurring accelerations have been measured at midspan. In the first test one jogger ran across the footbridge, while in the second test two joggers ran across the footbridge.

## 4. Discussions and conclusions

When calculating occurring accelerations due to joggers crossing a short span statically determined footbridge according to the SDOFM prescribed in guideline EUR 23984 EN, the results are considerably overestimated compared to what happens in reality. The analytical model, proposed in the paper, considering a harmonic oscillating point load moving forward in time, is a much better approximation of the actual occurring accelerations.

Considering the occurring accelerations a clear distinction can be made between vandalism and comfort. For vandalism one might use the SDOFM in the steady-state. In case of comfort the proposed analytical model is recommended.