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# FUZZY PROBABILISTIC METHOD OF FOOTBRIDGE VIBRATION SERVICEABILITY ASSESSMENT UNDER PEDESTRIAN LOADS

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#### Summary

In this paper, a vibration serviceability assessment method for normal unrestricted pedestrian stream condition is proposed adopting the theories of random vibration and fuzzy probabilistic reliability. Considering the randomness nature of pedestrian dynamic forces, the load model is build from the probabilistic perspective. With the spectral modal force, the probability density function of maximum dynamic response is estimated by random vibration theory. The human perception to vibration is sorted into five grades which can be regarded as different fuzzy sets and each has a corresponding membership function which is gained from a fuzzy statistic experiment. Then the probabilities of maximum dynamic response belonging to each perception grade are gained by applying the reliability theory. The whole procedure is finally illustrated by an application of a real footbdige.

**Keywords:** fuzzy probabilistic method; vibration serviceability assessment; pedestrian dynamic loads; vibration perception; shaking table test

#### 1. Introduction

In the area of human-induced vibrations of footbridges, a lot of research have been conducted. In the aspect of pedestrian forces, in order to represent more realistic pedestrian traffic in normal conditions, it is necessary to consider the intra- and inter- subject variability of pedestrian walking forces. As for the comfort criteria, it is generally known that different humans feel differently to the same vibration cases and even the individual exposed to the same vibration will probably feel differently in different moments. Thus, to evaluate the perception of vibrations, it is more reasonable to include the fuzzy essence of human perception.

In this paper, a comfort assessment method for normal unrestricted pedestrian stream condition is proposed with the combination of random vibration theory and fuzzy probabilistic reliability theory. Considering the randomness nature of pedestrian walking forces, the load model is build from the probabilistic perspective. And the human perception to vibrations (comfort grade) is classified into five levels which can be regarded as five fuzzy sets and each has a corresponding membership function which is gained from a fuzzy statistic experiment. An example is given in the last section. It should be noted that for simplicity only the vertical vibration case is elaborated. However, the method can also be applied to the horizontal case.



### 2. General Description of the Method

Each comfort grade can be regarded as a fuzzy set and has a corresponding membership function which can be gained from a fuzzy statistic experiment. Supposing that the fuzzy set is  $\theta$ , the maximum structural response is  $S_m$  and the membership function (MF) of  $S_m$  in fuzzy set  $\theta$  is  $\mu_{\theta}(x)$ , then by applying the fuzzy probabilistic reliability theory we can obtain the probability of  $S_m$  belonging to  $\theta$  as follow:

$$P_{\theta} = \int_{0}^{\infty} \mu_{\theta}(x) \cdot f_{S_{m}}(x) dx \tag{1}$$

where  $f_{S_m}(x)$  denotes the probability density function (PDF) of the maximum structural response, which can be gained from random vibration theory. Thus, to solve Eq.(1), it needs to get  $f_{S_m}(x)$  and  $\mu_{\theta}(x)$ .

## 3. PDF of Maximum Dynamic Response

The load model of pedestrians crossing the footbridge can be constructed from the probabilistic perspective. And further the power spectral density function (PSDF) of pedestrian dynamic loads can be introduced. With the spectral modal force, the maximum dynamic response can be estimated through the classic methods of linear random vibration theory. Finally the PDF of the maximum dynamic response  $f_{S_m}(x)$  for normal unrestricted pedestrian stream loading scenario is obtained.

$$f_{S_m}(x) = v_0 T \frac{x}{\sigma_X^2} \cdot \exp\left(-\frac{x^2}{2\sigma_X^2}\right) \exp\left(-v_0 T \cdot \exp\left(-\frac{x^2}{2\sigma_X^2}\right)\right)$$
(2)

## 4. Comfort Grades and Membership Functions

The experiment on the human perception of vibrations was conducted on a laboratory shaking table through which 22 test subjects walked. Based on the experimental data, the membership functions of the five fuzzy sets were obtained corresponding to the five predefined comfort grades (perception levels).



Figure 1. Membership frequencies (presented as dots) and fitted Membership functions  $\mu_{\theta_{i=1}\sim 5}(x)$  of peak acceleration  $a_{peak}$  in each fuzzy set  $\theta_{i=1}\sim 5}$  (presented as lines); f = 2.0Hz

# 5. Application of the Method

A real footbridge is analyzed as an example to demonstrate the application of the proposed method. The footbridge is a cable stayed bridge with main span of 262m and two inclined elliptical pylons. The 7<sup>th</sup> mode (vertical vibration), of which the frequency is 1.451Hz, is chosen to be the analyzed example. When 0.5pers/ $m^2$  pedestrian stream crosses the footbridge freely, it is of 86% possibility that the vertical vibration comfort grade of the bridge is "A - Excellent". As a qualitative evaluation, the result is sound and reasonable.

#### 6. Conclusion

A serviceability assessment method for footbridge under normal unrestricted pedestrian stream condition is proposed with the combination of random vibration theory and fuzzy probabilistic reliability theory. Considering the randomness nature of pedestrian walking forces, the load model is build from the probabilistic perspective. And the human perception to vibrations is analyzed from a fuzzy perspective with the help of an experiment. The research on more comprehensive and representative experiment and pedestrian stream load model should be further conducted to improve the method.