# The Calculating Method of Dynamic Analysis of Beams with Variable Cross-Section Based on the Transfer Matrix Method

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## Summary

In order to solve dynamic analysis of beams with variable cross-section more efficiently, taking a class of widely used beams as the breakthrough point, the calculation method was established based on the transfer matrix method. Firstly, taking a single-span beam as the basic element, the element transfer matrix of continuous beams with variable cross-section was derived by solving the partial differential equation of motion. Secondly, the boundary and continuity conditions at the supports were utilized to reduce the overall transfer matrix. Thirdly, the general algorithm for solving dynamic characteristics of continuous beams with variable cross-section was established to obtain the natural frequencies and the mode shapes. Finally, by employing New mark Beta Methods, dynamic analysis under moving loads was realized. And then, taking MATLAB as the platform, a numerical calculation program was developed to solve dynamic characteristics and dynamic analysis of beams with variable cross-section. The results show that, compared with the finite element method, the proposed method was verified to be accurate, simple, and efficient.

**Keywords:** Transfer matrix method; beams with variable cross-section; dynamic characteristics; dynamic analysis

## 1. Introduction

Beams with variable cross-section are widely used in the field of bridges engineering for their superior mechanical characteristics. With the rapid development of Chinese transportation infrastructure, researchers focused more attention on the bridge vibration. The finite element method provides a powerful tool for solving the bridge vibration. However, It is necessary to develop global dynamics equations and to assemble the total stiffness matrix so that it might cause too much computational cost or even overflow errors.

The transfer matrix method has been developed for a long time and been applied widely in structure mechanics and dynamics system [1]. When using the transfer matrix method, the global dynamics equations are unnecessary and the orders of involved matrices are decreased significantly. Thomson [2] applied the transfer method to more general vibration problems. Rubin S [3] provided a general treatment for transfer matrices and their relation to other forms of frequency response matrices. Rui Xiao-ting [4] took variable cross-section beams as multi stepped beams, which are found from some beam elements end-to-end, to solve the dynamic characteristics. He Bin [5] established the discrete time transfer matrix method by combining the finite element method and the discrete transfer matrix method, which was applied to the large deformation analysis and the dynamic stiffness of the model. These achievements have played an important role in science and engineering field.

However up to now, the transfer matrix is seldom applied in solving dynamic characteristics and dynamic analysis for continuous beams with variable cross-section. In this paper, firstly, based on