Rayleigh-Ritz/Finite Element Analysis Of Plates By Singularity Functions

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Abstract

A novel analysis method is developed for the analysis of Timoshenko type plate bending loaded by concentrated loads. A stiffness matrix and a consistent load vector are assembled via the minimum potential energy principle and solved for a set of unknown variables. The rules described herein for assembling such as matrix may yield a matrix that is rank deficient (the matrix column vectors do not span the space of dimension equal to the matrix size, there exists a null-space). Solution to the equations is possible by the use of the singular value decomposition, (s.v.d.). The original problem is thus transformed to one having less unknown variables that are all independent. The redundant transformed variables, that correspond to the null-space, are discarded. An industrial reinforced concrete raft on an elastic foundation is presented as a worked numerical example and compared to Strand7® software finite element analysis.

Keywords: reinforced concrete, slab, foundation, finite element analysis, Rayleigh-Ritz analysis

1 Introduction: Deflection Function For Plate Bending

In the case of plate bending, the vertical (z direction) deflection variable, w, is a function of the in plane Cartesian coordinates x and y, $w(x,y)$. We will deal with a rectangular plate of dimensions A and B, in the x and y directions respectively with the origin of the Cartesian coordinate system at the plate centre. For the convenience of applying the Gauss Quadrature integration rule, the following dimensionless position coordinates are introduced:

$$\xi = {}^{2x}/_{A} \tag{1}$$

$$\eta = {}^{2y}/_{B} \tag{2}$$

This definition maps the plate corners at the dimensionless coordinates (-1,-1), (1,-1), (1,1) and (-1,1) as shown in Figure 1. The deflection function for the plate is proposed to have the general form of:

$$w(\xi,\eta) = c_{i,j}\xi^{j-i}\eta^{i} + d_{k,l,m,n}\langle\psi(\xi,\eta)\rangle_{k,l,m,n} \tag{3}$$

And:

$$\langle\psi(\xi,\eta)\rangle_{k,l,m,0} = \langle\alpha_m - \xi\rangle^{l-k+3}\langle\eta - \beta_m\rangle^{k} \tag{4}$$

$$\langle\psi(\xi,\eta)\rangle_{k,l,m,1} = \langle\xi - \alpha_m\rangle^{l-k+3}\langle\eta - \beta_m\rangle^{k} \tag{5}$$

$$\langle\psi(\xi,\eta)\rangle_{k,l,m,2} = \langle\xi - \alpha_m\rangle^{l-k+3}\langle\beta_m - \eta\rangle^{k} \tag{6}$$

$$\langle\psi(\xi,\eta)\rangle_{k,l,m,3} = \langle\alpha_m - \xi\rangle^{l-k+3}\langle\beta_m - \eta\rangle^{k} \tag{7}$$

https://doi.org/10.2749/vancouver.2017.2937
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