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EFFECT OF PRETENSION ON THE DYNAMIC RESPONSE OF FOOTBRIDGES

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Summary

In this paper, we intend to study the effects of geometric nonlinearities on concrete footbridges. Dynamic characteristics of structures depend on their stiffness and mass. With those, we determine their natural frequencies and modes of free vibrations. Nevertheless, the initial stiffness of a structure, computed in its unloaded state, is affected by the applied forces, the so-called geometric stiffness. Compressive forces usually reduce the stiffness and the frequencies and may lead to buckling, for zero frequencies. In the other hand, traction loads tend to increase stiffness and frequencies, a phenomenon resorted upon by the so-called tensostructures. A class of structures of economic-strategic importance is footbridges, excited by vibrations induced by motions of persons. These vibrations may affect the structures but, in general, may render inadequate human walking conditions. There is a tendency of modern structural engineering towards slender members, due to more efficient materials and more powerful analysis tools. Here, we study these effects via a theoretical-numerical approach by an approximated model derived by Rayleigh's Method. The model is a concrete slab walkway under pretension supporting. We suppose the original design have provided for natural frequencies away from the excitation frequency. Nevertheless, the presence of large axial compressive force will reduce the beam stiffness and natural frequencies wich may lead to unexpected potentially dangerous resonance states.

Keywords: dynamics of structures; nonlinear dynamics of structures; geometric nonlinearity; pretension

1. Introduction and mathematical model

In this paper, we study the effects of geometric nonlinearities on footbridges designed as prestressed concrete slabs. Dynamic characteristics of structures depend on their stiffness. The initial stiffness of a structure, in its unloaded state, is affected by the applied forces, the so-called geometric stiffness. Compressive forces usually reduce the stiffness and the frequencies. These vibrations may affect resistance the structures, its ultimate limit state, but, in general, may render inadequate human walking conditions, a service limit state [1]. Here, we study these effects via an approximated model derived by Rayleigh's Method [2]. The model is a concrete slab under pretension designed as a walkway. We suppose the original design have provided for natural frequencies away from the excitation frequency. Nevertheless, the presence of large axial compressive force due to pretension will reduce the slab stiffness and natural frequencies that may lead to unexpected potentially dangerous resonance states, or, at least, uncomfortable walking conditions.

We analyzed a one meter wide strip of pre-stressed concrete slab reinforced only in its longitudinal direction. The span is *L,* equals to 8, 10 and 12m and thickness *h*, 16, 20 and 25cm, respectively. The concrete characteristic resistance is the same for all models, 35MPa, and elastic modulus 33130MPa.

The problem is reduced to a single degree of freedom using Raylegh's method [2], assming a shape function ϕ *(x)* so that the displacement is zero on the supports and assumes unitary value at mid section. Mass, elastic and geometric stiffness characteristics are given by

$$\phi(x) = sen\left(\frac{\pi x}{L}\right) \quad m^* = \overline{m}\int_0^L \phi^2 dx = \frac{\overline{m}L}{2} \quad k^* = EI\int_0^L (\phi'')^2 dx = \frac{\pi^4 EI}{2L^3} \quad k_G{}^* = P\int_0^L (\phi')^2 dx = \frac{P\pi^2}{2L} \qquad (1)$$

where m^* is the generalized mass, $\overline{m}$ the mass per unit length of the beam (considering the concrete density to be 2500 kg/m³), k^* is the generalized elastic stiffness, E the elastic modulus of concrete, I the section moment of inertia and $k_G{}^*$ the generalized geometric stiffness, dependent on the *P* axial compressive force. Next, acceleration responses of the structure are computed assuming a sinusoidal loading due to persons walking on the slab (2 Hz), and acceleration amplitude $\ddot{Z}$ for harmonic loading is

$$p^*(t) = \int_0^L p(t)\phi\, dx = \frac{2\overline{m_P}L^2}{\pi} \; sen(\Omega t) \qquad \ddot{Z} = \Omega^2 \frac{2\overline{m_P}L^2}{\pi \overline{k^*}} \frac{1}{\sqrt{(1-\beta^2)^2 + (2\xi\beta)^2}} \qquad (2)$$

2. Results

Table 1 lists frequency results (Hz) for the 3 proposed lengths, and respective thicknesses.

Table 1 – Frequencies and effective acceleration

L=8m and h=16cm				L=10m and h=20cm				L=12m and h=25cm			
σavg (MPa)	P (kN)	f (Hz)	aef (m/s²)	σavg (MPa)	P (kN)	f (Hz)	aef (m/s²)	σavg (MPa)	P (kN)	f (Hz)	aef (m/s²)
0	0	3.79	0.55	0	0	3.08	0.86	0	0	2.71	1.16
0.5	80	3.70	0.59	0.5	100	3.01	0.93	0.5	125	2.65	1.27
1.0	160	3.61	0.63	1.0	200	2.93	1.02	1.0	250	2.59	1.42
1.5	240	3.52	0.68	1.5	300	2.86	1.12	1.5	375	2.53	1.60
2.0	320	3.42	0.74	2.0	400	2.78	1.25	2.0	500	2.47	1.84
2.5	400	3.32	0.81	2.5	500	2.70	1.42	2.5	625	2.40	2.15
3.0	480	3.22	0.89	3.0	600	2.62	1.63	3.0	750	2.34	2.60
3.5	560	3.12	0.99	3.5	700	2.54	1.92	3.5	875	2.27	3.28
4.0	640	3.01	1.12	4.0	800	2.45	2.34	4.0	1000	2.20	4.43
4.5	720	2.90	1.28	4.5	900	2.36	2.98	4.5	1125	2.13	6.78
5.0	800	2.79	1.51	5.0	1000	2.26	4.10	5.0	1250	2.06	13.54
5.5	880	2.67	1.83	5.5	1100	2.17	6.56	5.5	1375	1.98	21.89
6.0	960	2.54	2.32	6.0	1200	2.06	15.24	6.0	1500	1.90	9.27

3. Conclusions

In this paper, we presented a model of a prestressed concrete slab walkway showing that the presence of the compressive axial loads affects resonance properties due to geometric nonlinearity.

4. Acknowledgements

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5. References

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