

## Ultimate spans and optimal rise relations of steel arches

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## 1 Abstract

Arched structures have been in use more than three thousand years. The span length of the arch in bridge construction has already exceeded 550 meters. Even longer arch bridges have been designed. The development of arch structures has not been as fast as in cable-stayed bridges during the recent 30 years, when cable-stayed bridges have reached spans more than 1100 m. However, arch structures are becoming more common, especially in bridges.

The purpose of this paper is to arouse interest in arch structures as well as to open up basic issues related to optimal arch design. The paper discusses the parabolic arch, the catenary arch, and the constant stress arch. The optimum heights, which produce the minimum amount of material, are solved. The importance of form-finding design is emphasized.

The optimum heights are solved mathematically. In addition, a non-linear iteration procedure, based on vector algebra solution, is used in finding the optimum shape of the moment-less arch. The applications of traditional graphic static and the usage of vector algebra are useful practical tools for designers, especially during the preliminary design stages.

The maximum theoretical arch spans are remarkably long. At a stress level of 500 MPa, the ultimate span of a steel arch is 19 635 m. The optimal heights of the arches are bigger than traditionally expected. For example, for a parabolic arch, the optimum span to height relation  $l/h$  is 2.309. It can also be mentioned that the optimum height ratio of 2.962, derived for the catenary arch, has so far been an unknown figure for designers.

The theoretical maximal dimensions resolved in the paper indicate that the dimensions of arches can be increased further.

**Keywords:** arches, form-finding, catenary arch, constant stress arch, parabolic arch, ultimate span