Combined Lower and Upper Bound Plastic Analysis: Better than the Sum of its Parts

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Summary

Studies have shown that current bridge assessments are often overly conservative. In particular, engineers often use elastic analysis to predict the load at which a concrete slab bridge will fail in flexure and thus may grossly underestimate its ultimate capacity. This leads to bridges being unnecessarily strengthened or even rebuilt, creating wasteful expense and unsustainable use of resources.

Upper bound plastic „yield line“ analysis is sometimes used to perform less conservative calculations, but it is difficult to apply to cases with complex geometry and engineers are often concerned that the result may an unsafe overestimate of the collapse load. Modern computational methods enable automated lower bound analysis, which has been found to give good results but is still not guaranteed to be accurate. This paper introduces the concept of „combined lower and upper bound plastic analysis“, in which the results of a lower bound analysis assist in generating an upper bound solution. The resulting close upper and lower bounds on the plastic collapse load prove that the maximum deviation from the theoretical exact solution is small.

Keywords: concrete; slab; yield line; plastic; lower bound; upper bound.

1. Introduction

Engineers are frequently asked to reassess existing concrete bridges to check that their load capacity remains adequate following an increase in loading, deterioration of the structure, or a change to the code requirements. Many assessments currently use conservative elastic analysis to predict the load at which concrete slab bridges collapse in flexure. This results in many bridges“ capacities being grossly underestimated, and may cause structures to be unnecessarily strengthened or replaced. Such unnecessary works are an unsustainable waste of resources, and may also cause disruption to traffic and hence increase congestion and pollution.

2. Plastic Analysis

Plastic analysis can be used to find a less conservative estimate of the flexural capacity of some concrete slab bridges. Traditionally, two forms of plastic analysis are available:

- Upper bound „yield line“ analysis can be performed by hand or with the aid of a computer for structures with simple geometries. However, the method is difficult to reliably apply to more complicated cases. Any errors are on the „unsafe“ side; they cause the engineer to overestimate the strength of the slab.
- Lower bound analysis has traditionally been difficult to use in assessment, but methods developed recently by the authors have helped to make its use more feasible. However, whilst there is anecdotal evidence that the method consistently produces safe yet accurate results, there is no guarantee that the results are close to the true collapse load.

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In many cases it is possible to conduct independent upper and lower bound analyses in order to fully bound the collapse load. However, for slabs with complex geometry, it is sometimes difficult to obtain bounds which are reasonably close.

3. Combined Lower and Upper Bound Analysis

In the method developed here for combined lower and upper bound plastic analysis, an engineer first performs an independent lower bound analysis. The resulting moment field is then used to identify locations and directions of possible yield lines. These are plotted in a diagram of „yield line indicators" (e.g. Fig. 1). An engineer can then identify a corresponding yield line pattern (Fig. 2), which should be close to the critical mechanism and will thus give an upper bound close to the original lower bound.

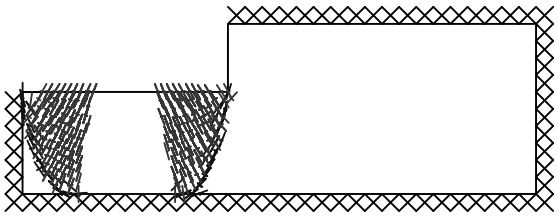

Fig. 1: Yield line indicators for the clamped slab

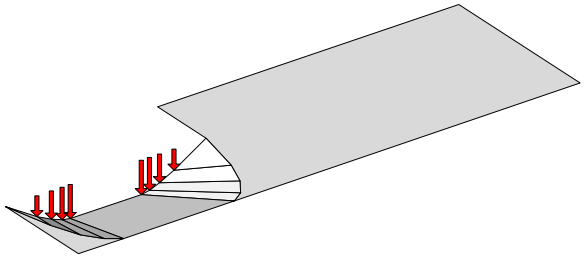

Fig. 2: Perspective view of new mechanism for the clamped slab

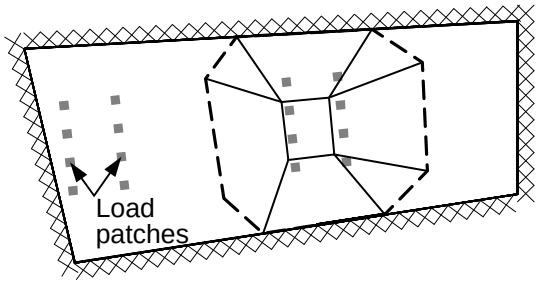

Fig. 3: Plan view of a real slab showing mechanism from independent upper bound analysis

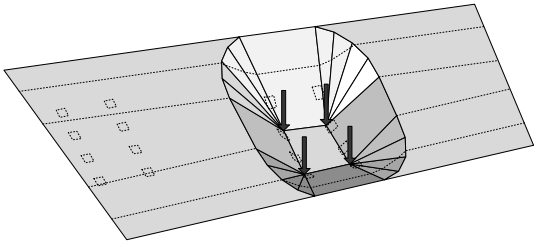

Fig.4: Perspective view of new mechanism for real slab

4. Examples

Two examples are presented. In the first, a uniformly loaded clamped slab (Fig. 1), the new method identifies a local mechanism (Fig. 2) not considered by a previous independent upper bound analysis. It finds close lower and upper bounds which bound the true collapse load more precisely than previous independent lower and upper bound analyses.

The second example is a real bridge slab which has previously been found inadequate by elastic analysis but was subsequently found adequate by an upper bound analysis (Fig. 3). The results of the new method (Fig. 4) agree closely with the previous independent upper bound analysis but offer a greater confidence that the result is correct.

Simplifications of the yield line patterns from both examples can be checked by hand to avoid any computational errors.

5. Conclusions

Combined lower and upper bound plastic analysis can be used to find lower and upper bounds to plastic flexural collapse loads. The results agree with previous analyses, but offer improvements:

- The derived bounds are closer than those identified by independent lower and upper bound analyses.
- The provision of both bounds gives engineers improved confidence in the accuracy of the results.
- The calculation of a critical mechanism enhances an engineer's ability to check the results using hand calculations.